Незатухающий ток в металлических и сверхпроводящих кольцах

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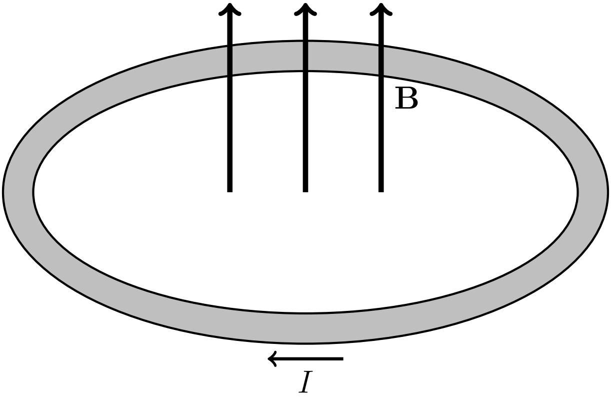
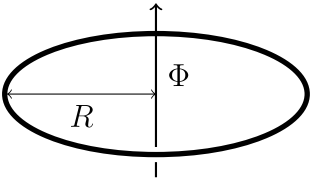
Problem specification



Outline

- Normal metal rings
  - Equilibrium
  - Adiabatic dynamics
  - Landau–Zener transitions
  - Dynamical localization
- Superconducting rings
  - Ginzburg–Landau model
  - Phase fluctuations in the thin rings

1D metal rings. Equilibrium



$$\psi_n = \frac{1}{\sqrt{2\pi}} e^{in\varphi}$$

$$E_n = \frac{\hbar^2 (n - \Phi/\Phi_0)^2}{2mR^2}$$

$$\Phi = \pi B R^2, \; \Phi_0 = \frac{2\pi\hbar c}{e}$$

$N = const$

$$E = \sum_{k=1}^{N} E_{n_k}$$

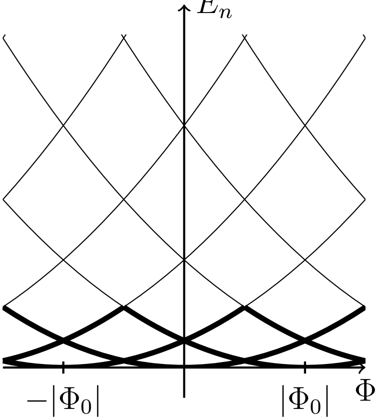
$\mu = const$

$$E = \sum_{E_n < \mu} E_n$$

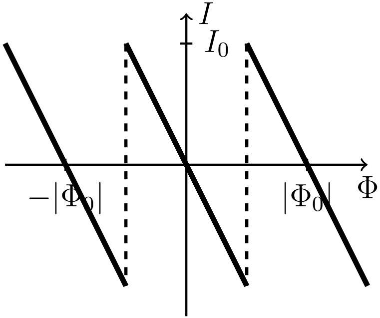
$$I = -c \frac{\partial E}{\partial \Phi}$$

Number of electrons $N \equiv 2 \pmod 4$

Energy spectrum



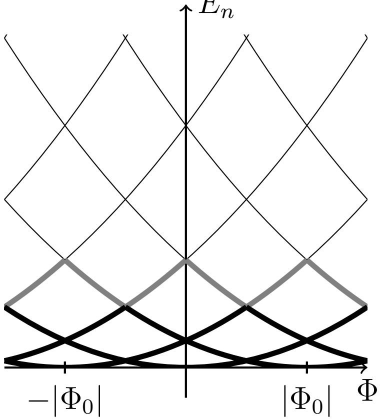
Current



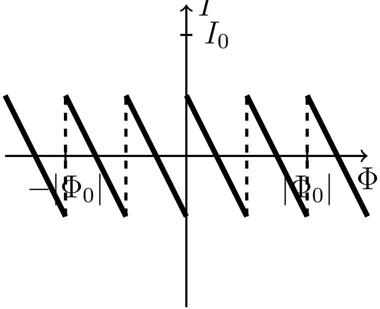
$$I_0 = -\frac{Ne\hbar}{4\pi m R^2} = -\frac{e v_F}{\pi R}$$

Number of electrons $N \equiv 3 \pmod 4$

Energy spectrum



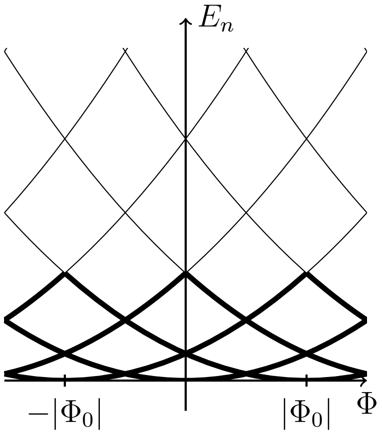
Current



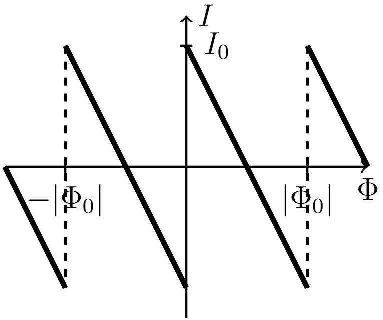
$$I_0 = -\frac{Ne\hbar}{4\pi m R^2} = -\frac{ev_F}{\pi R}$$

Number of electrons $N \equiv 0 \pmod 4$

Energy spectrum



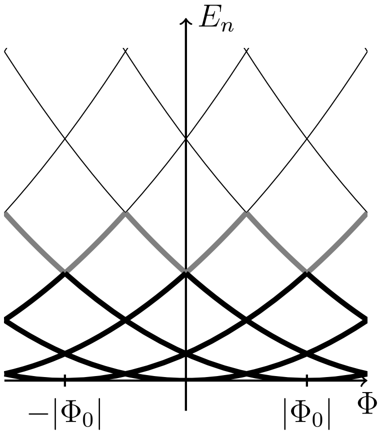
Current



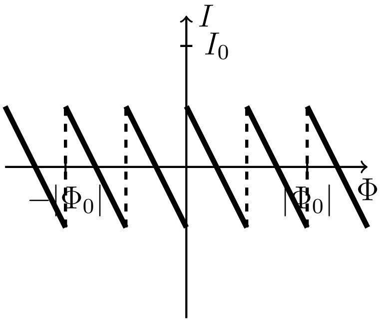
$$I_0 = -\frac{Ne\hbar}{4\pi m R^2} = -\frac{ev_F}{\pi R}$$

Number of electrons $N \equiv 1 \pmod 4$

Energy spectrum



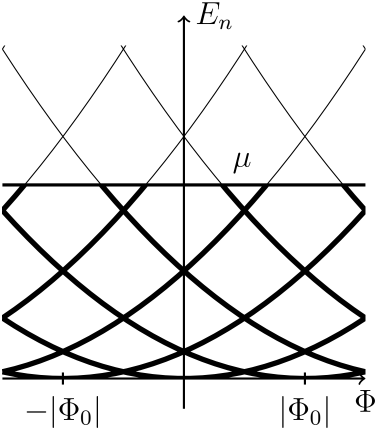
Current



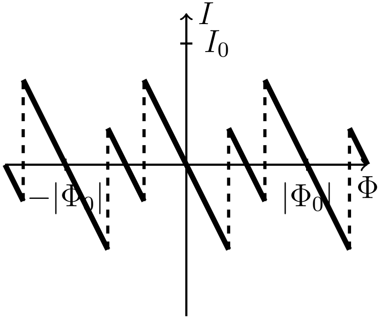
$$I_0 = -\frac{Ne\hbar}{4\pi m R^2} = -\frac{e v_F}{\pi R}$$

Chemical potential $\mu = const$

Energy spectrum



Current



$$I_0 = -\frac{Ne\hbar}{4\pi m R^2} = -\frac{ev_F}{\pi R}$$

Dynamics

Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2mR^2}\left(-i\frac{\partial}{\partial\varphi} - \frac{\Phi(t)}{\Phi_0}\right)^2\psi$$

Solution

$$\psi_n(\varphi,t) = \frac{1}{\sqrt{2\pi}}\exp\left(in\varphi - \frac{i}{\hbar}\int_0^t E_n(t')\,dt'\right)\ ,$$

$$E_n(t) = \frac{\hbar^2(n-\Phi(t)/\Phi_0)^2}{2mR^2}\ ,\ I(t) = \frac{2I_0\Phi}{\Phi_0}$$

Ideal diamagnetism!

Dynamics

Weak disorder

$$\hat{H} = \frac{\hbar^2(-i\partial_\varphi - \Phi/\Phi_0)^2}{2mR^2} + W(\varphi) \ ,$$

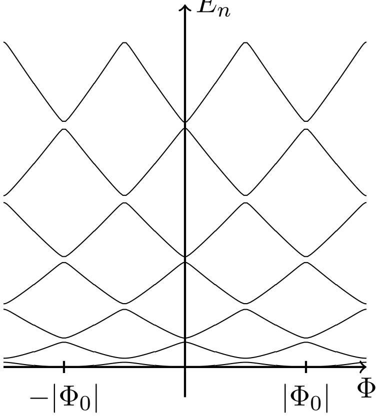
$$\hat{H}[\Phi]\Psi_n[\Phi] = E_n[\Phi]\Psi_n[\Phi]$$

Adiabatic dynamics

$$|\dot{\Phi}| \ll \frac{|\Phi_0(E_n - E_m)|}{\hbar}$$

$$\psi_n(t) = e^{i\chi(t)}\Psi_n[\Phi(t)]$$

Energy spectrum



Repulsion of the energy levels

Dynamics

Weak disorder

$$\hat{H} = \frac{\hbar^2(-i\partial_\varphi - \Phi/\Phi_0)^2}{2mR^2} + W(\varphi) \ ,$$

$$\hat{H}[\Phi]\Psi_n[\Phi] = E_n[\Phi]\Psi_n[\Phi]$$

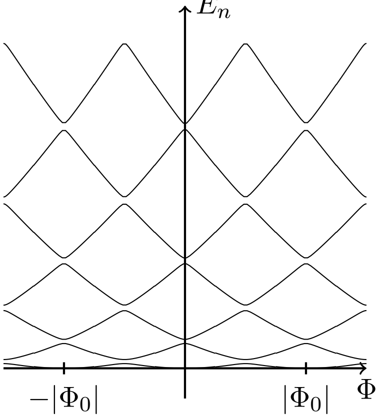
Adiabatic dynamics

Constant voltage $V = -\dot{\Phi}/c$

Current oscillates at frequency

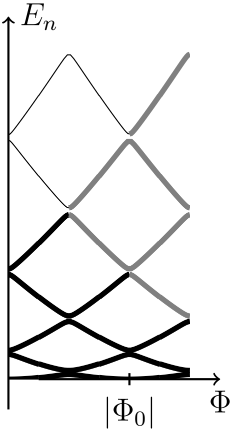
$\omega = eV/\hbar$

Energy spectrum



Repulsion of the energy levels

Landau–Zener transitions



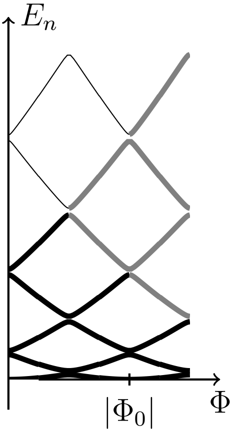
Wavefunction: $\psi(t) = \sum_n c_n(t)\Psi_n[\Phi(t)]$
Between transitions:

$$U_{nm}(t,t') = \delta_{nm} e^{-\frac{i}{\hbar}\int\limits_{t'}^{t} E_n[\Phi(t'')]\,dt''}$$

Transitions at half–integer Φ:

$$\hat{K}_h = \begin{pmatrix} \rho_1 & i\tau_1 & 0 & 0 & \dots \\ i\tau_1 & \rho_1 & 0 & 0 & \dots \\ 0 & 0 & \rho_2 & i\tau_2 & \ddots \\ 0 & 0 & i\tau_2 & \tau_2 & \ddots \\ \vdots & \vdots & \ddots & \ddots & \ddots \end{pmatrix}$$

Landau–Zener transitions



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Transitions at integer Φ:

$$\hat{K}_i = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & \tilde{\rho}_1 & i\tilde{\tau}_1 & 0 & 0 & \dots \\ 0 & i\tilde{\tau}_1 & \tilde{\rho}_1 & 0 & 0 & \dots \\ 0 & 0 & 0 & \tilde{\rho}_2 & i\tilde{\tau}_2 & \ddots \\ 0 & 0 & 0 & i\tilde{\tau}_2 & \tilde{\rho}_2 & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{pmatrix}$$

Dynamical localization

Evolution operator

$$\hat{\mathrm{U}} = \hat{K}_i \cdot \hat{U}(T/2) \cdot \hat{K}_h \cdot U(T/2),$$
$$T = 2\pi\hbar/(eV)$$

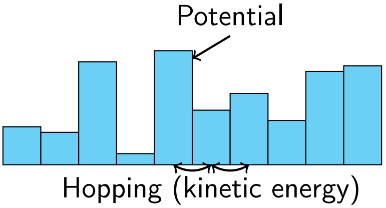
Quasienergy

$$\hat{\mathrm{U}}\mathbf{c}_k = e^{2\pi i T \epsilon_k/\hbar}\mathbf{c}_k$$

$\mathbf{c}_k$ are localized

$$\langle \hat{H} \rangle_{time} \to const$$
$$\langle \hat{I} \rangle_{time} \to 0$$

Anderson problem



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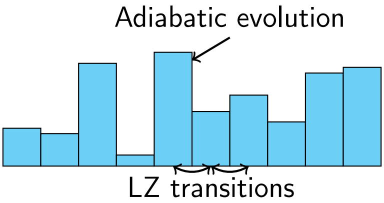


$$W_n^{eff} \sim \langle E_n \rangle (\mathrm{mod}\, 4\pi\hbar/T)$$

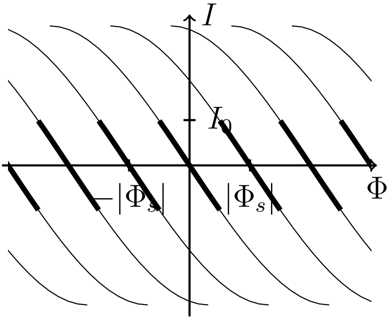
Superconducting rings. Ginzburg–Landau model

$$\psi = \psi_0 e^{in\varphi} \ ,$$

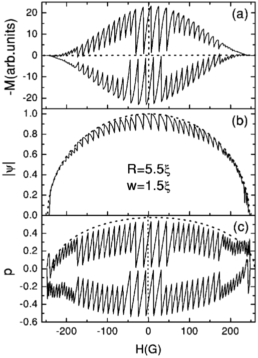
$$|\psi_0|^2 = 1 - \frac{\xi^2}{R^2}\left(n - \frac{\Phi}{\Phi_s}\right)^2 \ ,$$

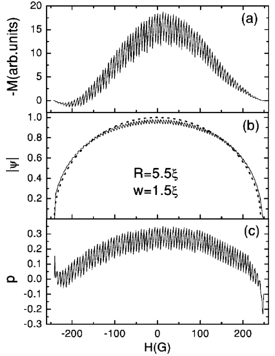
$$I = 2I_0|\psi_0|^2\left(n - \frac{\Phi}{\Phi_s}\right) \ ,$$

$$I_0 = \frac{cS\Phi_s}{8\pi^2\lambda^2 R}, \Phi_s = \frac{\Phi_0}{2}$$



Ginzburg–Landau model

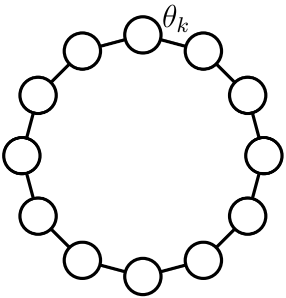




Vorticity jumps with the magnetic field[1]. Left: the homogeneous ring, right: the ring with the defect.

[1]D. Y. Vodolazov et. al., Phys. Rev. B **67**, 054506 (2003)

Quantum phase slips



Partition function

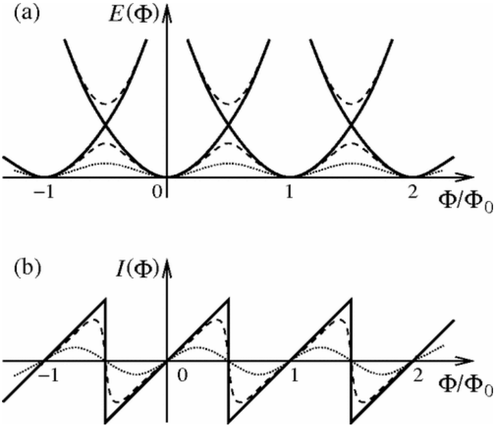
$$S = \int_0^\beta \sum_{k=1}^N \left\{ \frac{\dot{\theta}_k^2}{2E_C} + E_J \left[1 - \cos\left(\theta_k(\tau) - \phi\right)\right] \right\} d\tau$$

$$Z = \int D[\theta_k] e^{-S}, \ \sum_{k=1}^N \theta_k = 2\pi n, \ \phi = \frac{2\pi\Phi}{N\Phi_s}$$

Instantons (QPS)

$$\theta_k(\tau) = 4 \arctan \exp\left[\sqrt{E_J E_C}(\tau - \tau')\right]$$

Quantum phase slips



Energy and current dependence on the magnetix flux[2]

[2] K. A. Matveev, et. al., Phys. Rev. Lett. **89**, 096802 (2002)

Literature

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- R. Hübner and R. Graham, Phys. Rev. B **53**, 4870 (1996)
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- K. A. Matveev, et. al., Phys. Rev. Lett. **89**, 096802 (2002)

Thank you for attention!